Assignment 12.

1. It is given that the complex numbers r, s, and t are related by the equation

$$\frac{1}{r} = \frac{1}{s} + \frac{1}{t}.$$

If $s = 1 - 3\mathrm{i}$ and $t = 2 + \mathrm{i}$, find r in the form $x + y\mathrm{i}$ where $x, y \in \mathbb{R}$. [3]

2. Solve the simultaneous equations

$$\begin{cases} \mathrm{i}w + 3z = 2 + 4\mathrm{i} \\ w + (1-\mathrm{i})z = 2 - \mathrm{i} \end{cases},$$

expressing z and w in the form $x + y\mathrm{i}$, where x and y are real. [4]

3. On a single diagram, sketch the following loci and shade their common region: [4]

$$|z - 3 - 3\mathrm{i}| \leq 3, \qquad \text{and} \qquad |z - 1| \geq |z - \mathrm{i}|.$$

Hence obtain the maximum value of $|z - 5 - \mathrm{i}|$, where z satisfies both the conditions. [2]

4. Simplify $z = \dfrac{1+\sin\theta+\mathrm{i}\cos\theta}{1+\sin\theta-\mathrm{i}\cos\theta}$, and find its modulus and argument in terms of θ. [7]

5. (a) Simplify the expression $(6-3\mathrm{i})^2 - 8(2+3\mathrm{i})$. [2]

(b) Find the square roots of $11 - 60\mathrm{i}$, writing your answers in the form of $a + b\mathrm{i}$, where a and b are reals. [5]

(c) Hence solve the quadratic equation $2z^2 - (6-3\mathrm{i})z + 2 + 2\mathrm{i} = 0$. [3]

6. (†) The complex number z satisfies the equation $|z-1|+|z-3| = 2\sqrt{2}$. <u>Without writing z in the form of $x + y\mathrm{i}$</u>, show that $\operatorname{Re} z = \sqrt{2}|z-1|$. [4]

Total mark of this assignment: $30 + 4$.

The symbol (†) indicates a bonus question. Finish other questions before working on this one.